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WaterWorld: a self-parameterising, physically based model for application in data-poor but problem-rich environments globally

APPENDIX 1

WATERWORLD VERSION 1.0 EQUATIONS

This description of WaterWorld v1.0 summarises the detailed equation listing given in Mulligan & Burke (2005) and with the model at www.policysupport.org/waterworld. The model structure, time and spatial scales are described in the text so we cover only the equations here.

Temperature, dewpoint and liquid water content

Here we calculate the diurnal temperatures, dew point and liquid water content according to atmospheric inputs (see Table 1). *Key assumption: Cloud liquid water content (LWC) is proportional to absolute atmospheric humidity.* First, temperature is modified according to the diurnal temperature range as follows:

Hour 00:00–06:00 $T = \mathrm{Tmp} - (0.25 \times \mathrm{DTR})$
Hour 06:00–12:00 $T = \mathrm{Tmp}$
Hour 12:00–18:00 $T = \mathrm{Tmp} + (0.25 \times \mathrm{DTR})$
Hour 18:00–24:00 $T = \mathrm{Tmp}$

where Tmp = mean monthly temperature; T = mean monthly temperature for specified diurnal period; DTR = diurnal temperature range.

Dewpoint and vapour pressure are then calculated according to standard equations: $\mathrm{es} = \exp(26.66082 - 0.0091379024 \times (T + 273.15) - (6106.396/(T + 273.15)))$ where: T = temperature (°C); es = saturated vapour pressure (mb).

Air density and absolute humidity are calculated as: $\mathrm{AD} = (\mathrm{MSLP} \times 100)/((T + 273.15) \times 287)$, where: AD = Air-Density (kg/m^3), MSLP = mean sea level pressure (mb).

LWC varies linearly with absolute humidity (AH) under the assumption that the maximum AH observed at any one time is equivalent to the usually observed maximum LWC (0.0002 kg/m^3). Such a simplification is necessary because conversion of AH to LWC is complex depending on cloud condensation nuclei and cloud physics.

Dewpoint is calculated as: $b = 26.66082 - \ln(e)$; $\mathrm{Td} = ((b - \sqrt{((\mathrm{btemp}^{\wedge}2) - 223.1986)})/0.0182758048) - 273.15$ where Td = dewpoint (°C); e = vapour pressure (mb).

This means that the lifting condensation level (LCL) becomes: lcl_mb = (1/(((NT − Td)/223.15) + 1)^3.5) × MSLP and lcl_masl = max((44.3308 − 4.94654 × ((lcl_mb × 100)^0.190263)) × 1000.0) where NT = ground temperature (°C); lcl_mb = lifting condensation level (mb), lcl_masl = lifting condensation level (masl). Liquid water content is distributed rather simplistically as: LWC = (AH/max (AH)) × 0.0002 where max(AH) refers to the maximum observed AH in the tile at the current timestep.

Precipitation

Ground level cloud (fog) occurrence

Fog occurs where the ground altitude is greater than the LCL for the current location and timestep: fog = Dem > lcl, where Dem = elevation (m).

Forest edges

We first calculate the length of forest and emergent edges that act as a surface for impaction of fog. Key assumption: That forest edges are important and can be represented as catching surfaces. That, as in the Chiquito test sites (Mulligan & Burke 2005), there is a random directionality of forest edges.

Forest is given a one sided LAI = 3 and pasture LAI = 2. Forest edges are calculated according to the tree fractional cover. The empirical equation derived from Mulligan & Burke (2005, figure 59) provides the fractional forest edge length on the basis of tree fractional cover, this is converted to an actual length based on the cell size of the grid compared with the original Landsat grid cell size. The fraction of emergent tree height that is exposed is calculated as a 5% fraction of the area covered by tree. The division by four accounts for the fact that only one edge of a grid cell will face the wind from a particular direction.

$FEF = -3E-05 \times Tree \times 2 + 0.0036 \times Tree$ where $Tree =$ percentage tree cover, $FEF =$ forest edge fraction.

$FELEN = FEF \times ((Cs \times Cs)/(25 \times 25)) \times 100$ where $Cs =$ cell size (m), where $FELEN =$ forest edge length.

$EMELEN = (0.05 \times TreeFrac) \times ((Cs \times Cs)/(25 \times 25)) \times 100$, where $EMELEN =$ emergent edge length; $TreeFrac =$ fractional tree cover.

FELEN_FAC = (forestedgelenm/4), EMLEN_FAC = (emergentedgelenm/4), where FELEN_FAC, EMLEN_FAC = forest edge lengths facing a particular direction.

Fog deposition surface area

We now calculate the surface areas available for fog deposition on herbaceous and tree covers. Key assumption: The whole unshaded (one sided) leaf surface area of leaves is available for sedimentation (deposition). Fractional trapping areas for forest and pasture are calculated first (on the basis of leaf self -shading). These are then multiplied by the fractional covers of tree and pasture for the grid cell and the available LAI.

The surface area available for fog deposition (sedimentation) is thus calculated as: $FTS = (1 - (\exp((-0.7 \times 0.3 \times 10))))$; $PTS = (1 - (\exp((-0.7 \times 6 \times 0.5))))$ where FTS, PTS = forest trapping surface area (m/m) and pasture trapping surface area (m/m).

The total fractional surface area for deposition is then: $DFrac = (TreeFrac \times FTS \times TreeLAI) + ((1 - TreeFrac) \times PTS \times HerbLAI)$.

Wind speeds modified for exposure

We then modify wind speeds for the effects of topographic exposure. Exposure can be measured effectively from a DEM. Wind speeds are now modified for local wind direction dependent exposure using an approach modified from Ruel *et al*. (2002).

Key assumption: The empirical parameters determined by Ruel *et al*. (2002) (from wind tunnel studies) are representative.

Prec > 0: TRFI = WSP/DTV; Prec < =0: TRFI = 0, where Prec = precipitation (mm/hr), WSP = wind speed (m/s). DTV = drop terminal velocity (m/s).

Prec> 0: WSCF = 1 + Grad × TRFI × cos(ASP-WDIR); Prec < =0: WSCF = 0, where WSCF = wind, slope correction factor, Grad = tan slope gradient (°), ASP = aspect (°), WDIR = wind direction, Prec = monthly precipitation (mm).

WSCF = max(WSCF,0); Prec = Prec × WSCF.

Fog settling

We calculate the fog settling velocity according to Stokes Law, *based on the mean particle size for fog.*

Key assumption: That fog settling occurs under calm conditions and upwards fog turbulent diffusion is limited compared with this downward flux. $FSV = (980 \times ((7.5/10000)^2) \times (1 - 0.0013))/(18 \times 0.000185)$ where 7.5 = fog droplet size in μm.

Impaction fluxes

We calculate fog impaction fluxes of the basis of wind fluxes. The model has no spatial memory or budgeting of fog so fog passing through a forest is not necessary depleted along the flowpath – rather the model assumes that there is limitless availability of fog from the near surface atmosphere (when and where fog is present) thus no budget of atmospheric moisture is maintained. Key assumption: The wind speed reductions within forest and rough pasture measured at the FIESTA sites (Mulligan & Burke 2005) are generally representative. Impaction fluxes are calculated as:

WFL = (WSP × 3600) × EMLEN_FAC × 1.5; EMIMPFL = (LWC × WFL), where WFL = wind flux and EMIMPFL = emergent impaction flux.

Wind speed at the grid scale is assumed unaffected by passing through occasional emergents. 1.5 m is the average height of emergents above the surrounding canopy.

Finally the fog flux passing herbaceous cover is calculated using the correction for observed wind speeds at pasture heights and the height of pasture assumed to be 0.5 m. A fog inclination angle for fog inputs over forest and pasture is calculated, based on their respective wind speeds. A vertical flux is calculated as the fog settling velocity over the whole cell surface area (rather than the vertical catching surfaces used for impaction fluxes). The proportion of fog inputs that are deposited rather than impacted depends upon the cosine of the fog inclination angle over grassland and forest fractions.

$\mathrm{WFL} = (\mathrm{WSP} \times 0.5030 \times 3600) \times (1 - \mathrm{TreeFrac}) \times \mathrm{Cs} \times 0.5$; $\mathrm{GIFL} = (\mathrm{LWC} \times \mathrm{WFL})$, where GIFL = herbaceous cover impaction flux.

$\mathrm{FIAN} = \tan^{-1}((\mathrm{WSP} \times 0.6053)/\mathrm{FSV})$; $\mathrm{PIAN} = \tan^{-1}((\mathrm{WSP} \times 0.5030)/\mathrm{FSV})$, where FIAN = forest fog inclination angle, PIAN = pasture fog inclination angle, FSV = fog settling velocity (m/s according to Stokes Law).

$\mathrm{GFLUX} = (\mathrm{FSV} \times 3600) \times \mathrm{TAREA}$; $\mathrm{DPROP} = ((\cos(\mathrm{FIAN})) \times \mathrm{TreeFrac}) + \cos(\mathrm{PIAN})) \times (1 - \mathrm{TreeFrac}))$, where GFLUX = gravity flux, TAREA = cell true area (accounting for terrain slope), DPROP= proportion of fog flux that will be deposited.

$\mathrm{DIMP} = 1 - \mathrm{DPROP}$ where DIMP = proportion of fog flux that will be impacted.

Vegetation areas for fog interception

Next, the actual intercepting area of vegetation for fog is calculated because this will be combined with the previously calculated fog fluxes in order to calculate the actual fog interception. Surface areas for interception depend upon the leaf area density of the vegetation and the angle of incoming fog relative to leaves. Key assumption: Fog impaction occurs to all non shaded leaves according to the geometrical relationships between the angle of incoming fog (wind speed dependent) and the leaf area. Impaction only occurs on windward forest edges whereas fog passes over forest canopies or falls as deposition (sedimentation) on leeward (topographically sheltered) forests.

First, the forest trapping surface area is calculated as the self-shaded area of leaves exposed to fog droplets arriving at a particular angle (for the tree fraction of the cell). Pasture trapping surface area is calculated in a similar way (also according to pasture leaf area density and observed wind speeds). The impaction fraction is the fraction of the total potential impaction fluxes (to emergent tree, to tree edges and to herbaceous cover) that is trapped and so depends on the calculated forest and herbaceous trapping surface area. Importantly, impaction only occurs in the model when air is rising because the model assumes that air flows close to the ground when moving uphill (usually in windward exposed areas) but that fog flows above the ground in the leeward, more sheltered situations slopes, the parameter *AirRising* is true for situations where upwind elevation is greater than the downwind elevation. The equations are:

$\mathrm{FTSA} = (1 - (\exp((-0.7 \times 0.3 \times \mathrm{TreeFrac})/\cos(\mathrm{FIAN}))))$; $\mathrm{PTSA} = (1 - (\exp((-0.7 \times 6 \times (1 - \mathrm{TreeFrac}))/\cos(\mathrm{PIAN}))))$ where FTSA = forest trapping surface area and PTSA = pasture trapping surface area.

$\mathrm{IMFRAC} = (\mathrm{AirRising} \times \mathrm{FTSA})$; $\mathrm{IFLUX} = (\mathrm{EMIMPFL} + \mathrm{EdgeIFLUX} + \mathrm{GIFL})$; $\mathrm{SFLUX} = \mathrm{LWC} \times \mathrm{GFLUX}$, where IMFRAC = impaction fraction, AirRising indicates whether air is being moved up terrain or not, IFLUX = impaction flux (mm/hr) and SFLUX = deposition flux (mm/hr).

Ratio of impaction to sedimentation

The proportional flux that will be deposited compared with that which will be impacted is calculated. Key assumption: the balance between impaction and deposition depends upon the fluxes of water, the tendency towards lateral or vertical flow and the intercepting= areas for horizontal and vertical fluxes.

$\mathrm{DINT} = \mathrm{fog} \times (\mathrm{SFLUX} \times \mathrm{DPROP}) \times \mathrm{DFrac}$; $\mathrm{IINT} = \mathrm{fog} \times (\mathrm{IFLUX} \times \mathrm{DIMP}) \times \mathrm{IMFRAC}$, where DINT = intercepted deposition (kg/hr/cell) and IINT = intercepted impaction (kg/hr/cell), fog = potential presence of fog (site below LCL).

$\mathrm{FINT} = \mathrm{DINT} + \mathrm{IINT}$; $\mathrm{FINT\_mm} = (\mathrm{FINT}/\mathrm{TAREA}) \times (\mathrm{CFF})$, where FINT = total potential fog interception (kg/hr/cell), FINT_mm = total potential cloud interception (mm), CFF = fractional cloud frequency. Monthly total fluxes are the cumulation of the four monthly diurnal; fluxes and the 144 simulation hours that they represent.

The ‘flux’ is the volume of water passing by the representative surface area, the ‘frac’ is the fraction of that surface area that will intercept fog and the ‘prop’ is the proportion of the flux that is horizontal and vertical (dependent of the balance between local horizontal wind speed and settling velocity).The parameter ‘fog’ denotes areas above the LCL for that timestep so where there is no fog there will be no fog flux. The units of FINT, DINT and IINT are $kg/m^2/hr$. They are converted to mm/hr and multiplied by the cloud frequency to take account of those periods where the site may be above the LCL but no cloud generation has occurred.

Evapotranspiration

Radiation receipt and correction for cloud and fog

Extra terrestrial radiation receipts are generated according to standard equations (Iqbal 1983) are now converted to ground level radiation receipts by correction for dimming due to the presence cloud and fog. Key assumption: The radiation reductions observed under cloud and fog at the FIESTA sites (Mulligan & Burke 2005) are representative for other sites also.

fog = 1: $TL = (CFF \times 0.678) + ((1 - CFF) \times -0.143)$

fog = 0: $TL = (CFF \times 0.525) + ((1 - CFF) \times -0.143)$, where Tl = transmission loss (fraction).

$SMJ = SMJ \times (1 - TL)$, where SMJ = solar radiation (MJ).

The empirical parameters for the effect of fog and cloud on radiation receipts were calculated by comparison of measured radiation with modelled extraterrestrial radiation for a the FIESTA pasture site pixel in which the weather station sits (Mulligan & Burke 2005). The difference between modelled extraterrestrial and received land surface radiation by hour is a function of the transmission losses by cloud and fog. Thus these transmission losses were grouped according to those periods where the pasture site fog gauges were recording fog and those when they were not. This enabled the calculation of a mean transmission loss under cloudy conditions (no fog but $Rmeas \ll Rmodel$) and foggy conditions (fog present and $Rmeas \ll Rmodel$). Data were also analysed for clear conditions because the station recorded slightly lower values than the modelled values possibly because of more humid atmosphere above the station than parameterised in the atmospheric transmission component of the Iqbal (1983) solar radiation model.

Net radiation

Net radiation is calculated from a simple linear regression of net with solar radiation for sensors above a forest and a pasture cover (Mulligan & Burke 2005). Key assumption: The solar to net radiation conversion functions measured under forest and grassland are representative for larger areas and other covers of similar density that can be characterise as mixes of the Tree and Herb functional types.

$SWM = (SMJ \times 1000000)/(SIM/2)$.

$NWM = ((Tree/100) \times (-27.9 + (0.90 \times SWM)))$.

$NWM = NWM + ((1 - (Tree/100)) \times (-27.5 + (0.8 \times SWM)))$, where SIM = number of seconds in month, SWM = solar radiation input (W/m^2), NWM = net radiation (W/m^2).

Intercepted energy fractions

For simplicity and parsimony the model does not account for stomatal behaviour but rather defines the evapotranspiration differences between forest and pasture to be a function of the radiation intercepted by the canopy, since this is the driver of both transpiration and wet canopy evaporation. Key assumption: evapotranspiration can be effectively modelled at this coarse spatial and temporal scale from consideration of energy availablility and atmospheric demand for water only. Leaf area is sufficient to represent plant processes, and aerodynamic resistances can safely be ignored at these scales of analysis.

$ExpLAI = (1 - \exp(-0.7 \times \max(1, TreeLAI)))$.

$EtFrac = TreeFrac \times ExpLAI$.

$ExpLAI = (1 - \exp(-0.7 \times \max(1, HerbLAI)))$.

$EtFrac = EtFrac + ((1 - (TreeFrac + BareFrac)) \times ExpLAI)$.

Thus the overall intercepted energy for ET is the sum of energy intercepted by tree leaves and by herbaceous cover in the grid cell.

Evapotranspiration

Evapotranspiration is calculated on the basis of the energy available (the net radiation received) and the surface area available for transpiration and wet canopy evaporation. Because of the time and space scales used, surface, soil and wet canopy water balances were not possible so a water availability term could not be added to the model. Since available surface area (LAI) is a good surrogate for the availability of water through transpiring stomata or wet canopy evaporation, this was used here. Key assumption:

LAI can be used as a surrogate for water availability in determining evapotranspiration. The equations are:

$Ea = (611 \times \exp((17.27 \times NT)/(273.15 + NT)))/1000.$

$SSCK = (4098 \times Ea)/sqr(273.15 + NT).$

$PotE = (SSCK/(SSCK + 0.066)) \times NWM.$

$PotE = PotE \times (60 \times 60/1000000).$

$PotE > 0$: $PotE = (PotE/2.45)$; $PotE < =0$: $PotE = 0$;

$PotE > 0$: $ActE = PotE \times EtFrac$; $PotE < =0$: $ActE = 0$.

where NT = air temperature (°C), Ea = vapour pressure (KPa), SSCK = slope of the saturation vapour pressure curve (Kpa/C), NWM = Net radiation receipt (W/m^2) 2.45 = latent heat of vaporisation of water (MJ/kg).

Thus, evaporation is calculated on the basis of available energy and atmospheric demand to give potential evaporation and this is then combined with the non self-shaded surface area available for the interception of radiation/evaporation of water to give something closer to actual evaporation, which is responsive to vegetation type and cover as well as climate conditions. This approach is necessary in order to work with very differing vegetation and climate conditions over large spatial scales and at a monthly timestep where a full soil water balance is not possible.

Water balance

Water balance calculation

Key assumption: at these time and space scales losses to canopy, soil and groundwater are much less significant than the outcome of the fluxes of rainfall and evapotranspiration. Precipitation is converted from mm/month to mm/hr and the budget is calculated as:

$PR\_mm = Prec/(24 \times 30).$

$Budget = ((PR\_mm + FINT\_mm) - ActE).$

REFERENCES

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