**Supplementary materials**

Table S1 | Equations for the probability distribution functions used in this study

|  |  |
| --- | --- |
| Gumbel: (Fisher & Tippett 1928; Embrechts *et al*. 1997) | $$F\left(x\right)=exp\left\{-exp\left(-\frac{x-m}{α}\right)\right\}$$ |
| GEV: (Fisher & Tippett 1928; Embrechts *et al*. 1997) | $$F\left(x\right)=\left\{\begin{matrix}exp\left\{-\left[1-k\left(\frac{x-m}{α}\right)\right]^{{1}/{k}}\right\}\\exp\left\{-exp\left(-\frac{x-m}{α}\right)\right\}\end{matrix}\right.\begin{matrix}k\ne 0\\k=0\end{matrix}$$ |
| Gamma: (Weisstein 2016) | $$F\left(x\right)=\frac{1}{Γ\left(k\right)}γ\left(k,\frac{x}{α}\right)$$ |
| Pearson III: (US Water Resources Council 1982) | $$F\left(x\right)=\frac{1}{Γ\left(k\right)}γ\left(k,\frac{x-m}{α}\right)$$ |
| Generalized logistic: (Hosking & Wallis 1997) | $$F\left(x\right)=\left\{\begin{matrix}\left\{1+\left[1-k\left(\frac{x-m}{α}\right)\right]^{{1}/{k}}\right\}^{-1}\\\left\{1+exp\left(-\frac{x-m}{α}\right)\right\}^{-1}\end{matrix}\right.\begin{matrix}k\ne 0\\k=0\end{matrix}$$ |

Table S2 | Equations for the probability density functions used in this study

|  |  |
| --- | --- |
| Gumbel: (Fisher & Tippett 1928; Embrechts *et al*. 1997) | $$f\left(x\right)=\frac{1}{α}exp\left\{\left(\frac{x-m}{α}\right)\right\}exp\left\{-exp\left(-\frac{x-m}{α}\right)\right\}$$ |
| GEV: (Fisher & Tippett 1928; Embrechts *et al*. 1997) | $$f\left(x\right)=\left\{\begin{matrix}\frac{1}{α}\left[1-k\left(\frac{x-m}{α}\right)\right]^{{1}/{k-1}}exp\left\{-\left[1-k\left(\frac{x-m}{α}\right)\right]^{{1}/{k}}\right\}\\\frac{1}{α}exp\left\{\left(\frac{x-m}{α}\right)\right\}exp\left\{-exp\left(-\frac{x-m}{α}\right)\right\}\end{matrix}\right.\begin{matrix}k\ne 0\\k=0\end{matrix}$$ |
| Gamma: (Weisstein 2016) | $$f\left(x\right)=\frac{1}{α^{k}Γ\left(k\right)}\left(x\right)^{k-1}exp\left\{-\frac{x}{α}\right\}⁡$$ |
| Pearson type III: (US Water Resources Council 1982) | $$f\left(x\right)=\frac{1}{α^{k}Γ\left(k\right)}\left(x-m\right)^{k-1}exp\left\{-\frac{x-m}{α}\right\}⁡$$ |
| Generalized logistic: (Hosking & Wallis 1997) | $$f\left(x\right)=\left\{\begin{matrix}\frac{1}{α}\left[1-k\left(\frac{x-m}{α}\right)\right]^{{1}/{k-1}}\left\{1+\left[1-k\left(\frac{x-m}{α}\right)\right]^{{1}/{k}}\right\}^{-2}\\\frac{1}{α}exp\left\{\left(\frac{x-m}{α}\right)\right\}\left\{1+exp\left(\frac{x-m}{α}\right)\right\}^{-2}\end{matrix}\right.\begin{matrix}k\ne 0\\k=0\end{matrix}$$ |

Table S3 | Equations for the quantile functions used in this study

|  |  |
| --- | --- |
| Gumbel | $$x\left(F\right)=m-α\left[ln\left(-ln\left(F\right)\right)\right]$$ |
| GEV | $$x\left(F\right)=\left\{\begin{matrix}m+\frac{α}{k}\left[1-\left(1-ln\left(F\right)\right)^{k}\right]\\m-α\left[ln\left(-ln\left(F\right)\right)\right]\end{matrix}\right.\begin{matrix}k\ne 0\\k=0\end{matrix}$$ |
| Gamma | No explicit form, solved numerically (see Best & Roberts 1975) |
| Pearson type III | No explicit form, given in tables (US Water Resources Council 1982) |
| Generalized logistic | $$x\left(F\right)=\left\{\begin{matrix}m+\frac{α}{k}\left[1-\left\{{1}/{F}-1\right\}^{k}\right]\\m-αln\left\{{1}/{F}-1\right\}\end{matrix}\right.\begin{matrix}k\ne 0\\k=0\end{matrix}$$ |

**Table S4** | The ordinary moment estimates for selected distributions (see, e.g., Bezak *et al*. 2014)

|  |  |  |
| --- | --- | --- |
|  | **Moments** | **Parameters** |
| Gumbel: (Stedinger *et al*. 1993) | $$\begin{matrix}μ=m+α\*0.577\\σ^{2}=\frac{π^{2}}{6}α^{2}\end{matrix}$$ | $$\begin{matrix}μ=m+α\*0.577\\α=σ\frac{\sqrt{6}}{π}\end{matrix}$$ |
| GEV: (Stedinger *et al*. 1993) | $$\begin{matrix}μ=m+\frac{α}{k}\left(1-Γ\left(1+k\right)\right)\\σ=\frac{α}{k}\sqrt{\left(Γ\left(1+2k\right)-Γ^{2}\left(1+2k\right)\right)}\\C\_{s}=sgn(k)\frac{-Γ\left(1+3k\right)+3Γ\left(1+k\right)Γ\left(1+2k\right)-2Γ^{3}\left(1+k\right)}{\left[Γ\left(1+2k\right)-Γ^{2}\left(1+2k\right)\right]^{{3}/{2}}}\end{matrix}$$ | $$\begin{matrix}m=μ-\frac{α}{k}\left(1-Γ\left(1+k\right)\right)\\α=\frac{σ|k|}{\sqrt{\left(Γ\left(1+2k\right)-Γ^{2}\left(1+2k\right)\right)}}\\k:solved numerically\end{matrix}$$ |
| Gamma: (Weisstein 2016) | $$\begin{matrix}μ=αk\\σ^{2}=kα^{2}\end{matrix}$$ | $$\begin{matrix}α={σ^{2}}/{μ}\\k={μ^{2}}/{σ^{2}}\end{matrix}$$ |
| Pearson type III: (Basak & Balakrishnan 2012) | $$\begin{matrix}μ=m+αk\\σ^{2}=kα^{2}\\C\_{s}=\frac{2}{\sqrt{k}}\end{matrix}$$ | $$\begin{matrix}m=μ-2\frac{σ}{C\_{s}}\\α=\frac{1}{2}σ\left|C\_{s}^{}\right|\\k=\frac{4}{C\_{s}^{2}}\end{matrix}$$ |
| Generalized logistic: (Shin *et al*. 2009) | $$\begin{matrix}μ=m+\frac{α}{k}\left(1-g\_{1}\right)\\σ^{2}=\frac{α^{2}}{k^{2}}\left(g\_{2}-g\_{1}^{2}\right)\\C\_{s}=\frac{k}{\left|k\right|}\frac{k-g\_{3}+3g\_{1}g\_{2}-2g\_{1}^{3}}{\left(g\_{2}-g\_{1}^{2}\right)^{{3}/{2}}}\end{matrix}$$ | $$\begin{matrix}m=μ-\frac{α}{k}\left\{1-Γ\left(1+k\right)Γ\left(1-k\right)\right\}\\α=sign(k)\frac{σk}{\left\{g\_{2}-g\_{1}^{2}\right\}^{{1}/{2}}}\\k≈\frac{2}{3π} tan^{-1}\left(-0.59484C\_{s}\right)\end{matrix}$$ |

\*$g\_{r}=Γ\left(1+rk\right)Γ\left(1-rb\right)$, Г() is the gamma function and *gr* existst only if *β* < 1/r.

**Table S5** | The linear moment estimates for selected distributions (see, e.g., Bezak *et al*. 2014)

|  |  |  |
| --- | --- | --- |
|  | **L-moments** | **Parameters** |
| Gumbel: (Hosking & Wallis 1997) | $$\begin{matrix}λ\_{1}=m+α\*0.5772\\λ\_{2}=α\*ln\left(2\right)\end{matrix}$$ | $$\begin{matrix}m=λ\_{1}-\frac{0.5772\*λ\_{2}}{ln2}\\α=\frac{λ\_{2}}{ln2}\end{matrix}$$ |
| GEV: (Hosking & Wallis 1997) | $$\begin{matrix}λ\_{1}=m+\frac{α}{k}\left(1-Γ\left(1+k\right)\right)\\λ\_{2}=\frac{α}{k}\left(1-2^{-k}Γ\left(1+k\right)\right)\\τ\_{3}=2{\left(1-3^{-k}\right)}/{\left(1-2^{-k}\right)}-3\end{matrix}$$ | $$\begin{matrix}m=λ\_{1}-\frac{α}{k}\left(1-Γ\left(1+k\right)\right)\\α=\frac{λ\_{2}k}{\left(1-2^{-k}Γ\left(1+k\right)\right)}\\k≈7.8590c+2.995c^{2}\_{}c=\frac{2}{3+τ\_{3}}-\frac{log2}{log3}\end{matrix}$$ |
| Gamma: (Hosking 1990) | $$\begin{matrix}λ\_{1}=αk\\λ\_{2}=\frac{1}{\sqrt{π}}α\frac{Γ\left(k+0.5\right)}{Γ\left(k\right)}\\\end{matrix}$$ | $$\begin{matrix}α=\frac{λ\_{1}}{k}\\k=\begin{matrix}\frac{1-0.3080z'}{z'-0.05812z'^{2}+0.01765z'^{3}}&t<0.5\end{matrix}\\k=\frac{0.7213z^{''}-0.5947z''^{2}}{1-2.1817z^{''}+1.2113z''^{2}}\begin{matrix}&0.5 t<1.0\end{matrix}\end{matrix}$$$$\begin{matrix}\begin{matrix}t=\frac{λ\_{1}}{λ\_{2}}&z'=πt^{2}\end{matrix}&z^{''}=1-t&\end{matrix}$$ |
| Pearson type III: (Hosking 1990) | $$\begin{matrix}λ\_{1}=m+αk\\λ\_{2}=\frac{α}{\sqrt{π}}\frac{Γ\left(k+0.5\right)}{Γ\left(k\right)}\\τ\_{3}=2{\left(1-3^{-k}\right)}/{\left(1-2^{-k}\right)}-3\end{matrix}$$ | $$\begin{matrix}\begin{matrix}m= λ\_{1}-αk\\α=λ\_{1}\sqrt{π}\frac{Γ\left(k\right)}{Γ\left(k+0.5\right)}\end{matrix}\\k=\begin{matrix}\frac{1+0.2906z'}{z^{'}+0.1882z'^{2}+0.0442z'^{3}}&|τ\_{3}|<0.3\end{matrix}\\k=\frac{0.36067z^{''}-0.59567z^{''}^{2}+0.25361z''^{3}}{1-2.78861z^{''}+2.56096z^{''}^{2}-0.77045z''^{3}}\begin{matrix}&0.3 <|τ\_{3}|<1.0\end{matrix}\end{matrix}$$$$\begin{matrix}\begin{matrix}&z'=3πτ\_{3}^{2}\end{matrix}&z^{''}=1-τ\_{3}&\end{matrix}$$ |
| Generalized logistic | $$\begin{matrix}λ\_{1}=m+α\left(\frac{1}{k}-\frac{π}{sin\left(kπ\right)}\right)\\λ\_{2}=\frac{αkπ}{sin\left(kπ\right)}\\τ\_{3}=-k\end{matrix}$$ | $$\begin{matrix}m=λ\_{1}-α\left(\frac{1}{k}-\frac{π}{sin\left(kπ\right)}\right)\\α=\frac{λ\_{2}sin\left(kπ\right)}{kπ}\\k=-τ\_{3}\end{matrix}$$ |

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