

Comparing grey formulations of the velocity-area method and entropy method for discharge estimation with uncertainty

APPENDIX A

The maximum entropy principle (POME) is used as statistical inference to solve a probability matter (Singh 1986, 2011), for instance to select a probability distribution function, $p(v)$, when the information available about the variable, v (in this case the flow velocity) is limited to some average quantities and defined constraints, such as the mean, variance, etc.

In short, entropy was first expressed by Shannon (1948) and was applied by Chiu (1987) for open channels:

$$H(v) = - \int_0^{v_{\max}} p(v) \ln p(v) dv \quad (30)$$

with $p(v)$ must satisfy two constraints, i.e.,

$$\int_0^{v_{\max}} p(v) dv = 1 \quad (31)$$

$$\int_0^{v_{\max}} vp(v) dv = \bar{v} \quad (32)$$

To maximize $H(v)$, subjected to the two above constraints, we need to solve (Singh 1986):

$$\frac{\partial(p(v) \ln p(v))}{\partial p(v)} + \lambda_1 \frac{\partial p(v)}{\partial p(v)} + \lambda_2 \frac{\partial(vp(v))}{\partial p(v)} = 0 \quad (33)$$

where λ_i are Lagrange multipliers. Equation (33) gives:

$$p(v) = e^{(\lambda_1 - 1)} e^{\lambda_2 v} \quad (34)$$

Therefore, replacing Equation (34) in the first constraint, Equation (31), and assuming $M = \lambda_2 v_{\max}$ we obtain (Chiu 1987):

$$p(v) = \frac{e^M}{v_{\max}(e^M - 1)} e^{\frac{M}{v_{\max}}v} \quad (35)$$

Finally, substituting Equation (35) into Equation (32) provides:

$$\frac{\bar{v}}{v_{\max}} = \frac{e^M}{e^M - 1} - \frac{1}{M} = \Phi(M) \quad (36)$$

Equation (36) shows the relationship between mean and maximum flow velocity, which Xia (1997) found perfectly linear along both straight reaches and river bends for the Mississippi River in the USA. A similar trend was obtained in gauged sections of different rivers across the world (Chen & Chiu 2004; Moramarco *et al.* 2004, 2011; Ardicioglu *et al.* 2005, to cite just a few).

APPENDIX B

Let $W_{v-a,i}$, the width of the i th grey number of discharge estimated by the velocity-area method, that is

$$W_{v-a,i} = Q_{v-a,i}^+ - Q_{v-a,i}^- \quad (37)$$

$Q_{v-a,i}^-$ and $Q_{v-a,i}^+$ being the lower and upper boundaries, respectively, of the i th grey discharge estimated through the velocity-area method; similarly, let $W_{e,i}$ the width of the i th grey number of discharge estimated by the entropy method, that is

$$W_{e,i} = Q_{e,j}^+ - Q_{e,i}^- \quad (38)$$

$Q_{e,j}^-$ and $Q_{e,j}^+$ being the lower and upper boundaries, respectively, of the i th grey discharge estimated through the entropy method. The mean absolute error width AWE (absolute width error) is defined as

$$AWE = \frac{1}{n_{\text{obs}}} \sum_{i=1}^{n_{\text{obs}}} |W_{v-a,i} - W_{e,j}| \quad (39)$$

while the average percentage error on the amplitude PWE (percentage width error) is defined as

$$PWE = \frac{1}{n_{\text{obs}}} \sum_{j=1}^{n_{\text{obs}}} \frac{|W_{v-a,j} - W_{e,j}|}{W_{v-a,j}} \quad (40)$$

The average percentage error on the lower boundary estimate of the grey numbers of the discharge, PQ^-E , is defined as

$$PQ^-E = \frac{1}{n_{\text{obs}}} \sum_{j=1}^{n_{\text{obs}}} \frac{|Q_{v-a,j}^- - Q_{e,j}^-|}{Q_{v-a,j}^-} \quad (41)$$

Similarly, the average percentage error on the upper boundary estimate of the grey numbers of the discharge, PQ^+E , is defined as

$$PQ^+E = \frac{1}{n_{\text{obs}}} \sum_{j=1}^{n_{\text{obs}}} \frac{|Q_{v-a,i}^+ - Q_{e,j}^+|}{Q_{v-a,i}^+} \quad (42)$$

REFERENCES

- Ardiclioglu, M., de Araújo, J. C. & Senturk, A. I. 2005 [Applicability of velocity distribution equations in rough-bed open-channel flow](#). *La Houille Blanche* **4**, 73–79.
- Chen, Y. C. & Chiu, C. L. 2004 [A fast method of flood discharge estimation](#). *Hydrol. Process.* **18** (9), 1671–1684.
- Chiu, C. L. 1987 [Entropy and probability concepts in hydraulics](#). *J. Hydr. Engrg. ASCE* **113** (5), 583–600.
- Moramarco, T., Saltalippi, C. & Singh, V. P. 2004 [Estimation of mean velocity in natural channels based on Chiu's velocity distribution equation](#). *J. Hydrol. Eng.* **9** (1), 42–50.
- Moramarco, T., Saltalippi, C. & Singh, V. P. 2011 [Velocity profiles assessment in natural channel during high floods](#). *Hydrol. Res.* **42** (2–3), 162–170.
- Shannon, C. E. 1948 [A mathematical theory of communication](#). *Bell System Tech. J.* **27** 379–423, 623–656.
- Singh, V. P. 1986 [Derivation of some frequency distributions using the principle of maximum entropy \(POME\)](#). *Adv. Water Res.* **9**, 91–106.
- Singh, V. P. 2011 [Hydrologic synthesis using entropy theory: review](#). *J. Hydrol. Eng.* **16**, 421–433.
- Xia, R. 1997 [Relation between mean and maximum velocities in a natural river](#). *J. Hydrol. Eng.* **123** (8), 720–723.