The Polluter Pays Principle as a policy tool in an externality model for nitrogen fertilizer pollution

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Appendix

A. Mathematical modeling of the ‘private’ solution of the profit maximizing farmer

The optimization problem faced by the farmer presented in Section 3.1 is to maximize its private net benefit ($\Pi_p$) and is given by:

$$\text{MAX } \Pi_p(N_{fert}(t), W_{ir}(t)) = P_Y \cdot Y(N_{fert}(t), W_{ir}(t)) - \text{TPC}(N_{fert}(t), W_{ir}(t))$$  \hspace{1cm} (A1)

Solving Equation (A1) for $N_{fert}(t)$ and $W_{ir}(t)$ with the aid of the explicit equations presented in Section 3.1 results in the first-order conditions for the ‘private’ solution of a profit-maximizing farmer:

$$\frac{\partial \Pi(N_{fert}(t), W_{ir})}{\partial N_{fert}(t)} = 0 \Leftrightarrow P_Y \cdot (a_1 - 1.5 \cdot a_3 \cdot (N_{fert}(t))^{0.5} - a_5 \cdot W_{ir}(t)) = P_N$$  \hspace{1cm} (A2)

$$\frac{\partial \Pi(N_{fert}(t), W_{ir})}{\partial W_{ir}(t)} = 0 \Leftrightarrow P_Y \cdot (a_2 - 1.5 \cdot a_4 \cdot (W_{ir}(t))^{0.5} - a_5 \cdot N_{fert}(t)) = P_W$$  \hspace{1cm} (A3)

Equations (A2) and (A3) represent economic efficiency in perfect competition, where the value of the marginal product equals that of the input price. Solving (A2) and (A3) results in the optimal amount of
irrigation water \((W_{ir}^{of})\) and nitrogen fertilizer \((N_{fen}^{of})\):

\[
N_{fen}^{of} = \frac{P_Y \cdot (a_1 - a_5 \cdot W_{ir}^{of}) - P_N}{1.5 \cdot P_Y \cdot a_3} \quad (A4)
\]

\[
W_{ir}^{of} = \frac{P_Y \cdot (a_2 - a_5 \cdot N_{fen}^{of}) - P_W}{1.5 \cdot P_Y \cdot a_4} \quad (A5)
\]

Equations (A4) and (A5) are solved for given prices \(P_Y, P_N\) and \(P_W\). We can set different fixed allocations of irrigation water for different prices of water, i.e. \(\tilde{W}_{ir} = W_{ir}^{of}\) for a given \(P_W\). We can then use these water allocations in the simulation of the model (in Section 4.2 above).

B. Mathematical modeling of the cost of treating groundwater contamination

The total cost of treating groundwater contamination (TTC) is paid by the drinking water consumers. This cost depends on the price they pay per m\(^3\) of drinking water after treatment, bringing water quality to health standard concentration \((P_{\tilde{Q}_{ur}})\), where \(\tilde{Q}_{ur}\) is the health standard nitrogen concentration for drinking water, and the treated quantity of drinking water to be supplied \(\tilde{W}_{ur}\) where \(i = 1, 2\) are the primary and primary plus secondary treatment stages, respectively. A general treatment cost function is given by: \(\text{TTC}(\tilde{Q}_{ur}, \tilde{W}_{ur}) = P_{\tilde{Q}_{ur}} \cdot \tilde{W}_{ur}\). Empirical data shows a linear relationship between \(P_{\tilde{Q}_{ur}}\) and \(Q(t)\) such that: \(P_{\tilde{Q}_{ur}}(Q(t)) = a + b \cdot Q(t); \quad a, b > 0\). The calculation of \(P_{\tilde{Q}_{ur}}\) depends on four main constituent costs (Ooku & Abir, 2000): capital cost, operating cost, pumping cost (cost of diverting the treated water at the right pressure to the municipal water network), and the post-treatment concentration removal cost to the municipal sewage system. The last two types of costs do not depend upon the nitrogen concentrations in the raw water and therefore do not depend on the number of treatment stages. Hence, in order to avoid double counting when more than one treatment stage is applied, the associated costs are considered only at the primary treatment stage. We do this because the marginal cost of adding a second stage includes only capital costs and the operating costs of the second plant. We define \(0 < \theta < 1\) as the ratio of capital and operating costs to total treatment cost per m\(^3\) of a representative treatment plant, and multiply this ratio by the price of the secondary treatment stage. We define \(0 < \theta < 1\) as the ratio of capital and operating costs to total treatment cost per m\(^3\) of a representative treatment plant, and multiply this ratio by the price of the secondary treatment stage. The total price paid per m\(^3\) of drinking water after primary plus secondary treatment stages is \(P_{\tilde{Q}_{ur}}(Q(t)) = P_{\tilde{Q}_{ur1}} + \theta \cdot P_{\tilde{Q}_{ur2}}\), where \(P_{\tilde{Q}_{ur1}} = a + b \cdot (\tilde{Q}_{ur}/v)\) is the price for maximum nitrogen concentration, \((\tilde{Q}_{ur}/v)\), in the primary treatment stage, \(v\) is a fixed rate of nitrogen concentration residual in the water after the treatment and \(P_{\tilde{Q}_{ur2}} = a + b \cdot (Q(t) - (\tilde{Q}_{ur}/v))\) is the price in the secondary stage. Rearranging the above equations gives a discontinuous total cost function of treating groundwater contamination (TTC) with three segments:

\[
\text{TTC} = \begin{cases} 
0; & \text{if } 0 \leq Q(t) \leq \tilde{Q}_{ur} \\
(a + b \cdot Q(t)) \cdot \tilde{W}_{ur1}; & \text{if } \tilde{Q}_{ur} < Q(t) \leq \frac{\tilde{Q}_{ur}}{v} \\
(c + d \cdot Q(t)) \cdot \tilde{W}_{ur2}; & \text{if } \frac{\tilde{Q}_{ur}}{v} < Q(t) \leq \frac{\tilde{Q}_{ur}}{v^2}
\end{cases}
\] (B1)
This discontinuous cost function reflects the reality better than the continuous one. The treatment cost, up to the environmental standard ($\bar{Q}_{ur}$), is zero. But when passing this standard, even marginally, there is a sudden jump due to the investment in a treatment plant. In that case, and up to a concentration of ($\bar{Q}_{ur}/v$), the cost of treating groundwater contamination is $(a + b \cdot Q(t)) \cdot \bar{W}_{ur1}$. This is also true regarding the decision to invest in one stage or two stage treatment plants and is crucially dependent on the capital cost. If a second treatment is needed, then the cost of treating groundwater contamination jumps to $(c + d \cdot Q(t)) \cdot \bar{W}_{ur2}$ because the third segment is higher than the second segment of Equation (B1) where $c = a \cdot (1 + \theta) + b \cdot \bar{Q}_{ur}/v \cdot (1 - \theta)$ and $d = \theta \cdot b$. Equation (B1) still represents constant return to scale at each segment that a positive cost applies. However, there are different slopes (different marginal costs) between the second and third segment ($b > d$ because $0 < \theta < 1$). This could represent an increasing return to scale (or a decreasing average cost function where the two segments are combined by a continuous function). We assume $\bar{W}_{ur1}$ and $\bar{W}_{ur2}$ to be constant (not dependent on $Q(t)$); therefore Equation (B1) represents the effect of $Q(t)$ on TTC only through $P_{\bar{Q}_{ur}}(Q(t))$. We set $\bar{W}_{ur1}$ and $\bar{W}_{ur2}$ to be equal to a constant share $(1 - z)$ of drinking water before the treatment process ($\bar{W}_{ur1} = \bar{W}_{ur2} = (1 - z) \cdot \bar{W}_{ur}$), where the share $(1 - z)$ is the final water recovery share of the primary treatment stage with nitrogen concentration equal $\bar{Q}_{ur}/v$. Equation (B1) is demonstrated graphically in Figure B1 where (a) represents total treatment costs and (b) represents marginal treatment costs.

C. Mathematical modeling of solving the possible points ($N_{fert1}^{obs} \cdot Q_{obs1}$) and ($N_{fert3}^{osp} \cdot Q_{osp3}$) of the optimal social benefit, which fall where the treatment cost function is discontinuous and is not differential

The possible points ($N_{fert1}^{obs} \cdot Q_{obs1}$) and ($N_{fert3}^{osp} \cdot Q_{osp3}$) of the optimal social benefit, which fall where the treatment cost function is discontinuous and is not differential are solved with a strict restriction.
on the farmer, namely to use quantities of nitrogen fertilizer equal to $N_{fert}^{osb1}$ or $N_{fert}^{osb3}$. By using these quantities, the nitrogen concentration in the groundwater converges to $Q_{osb1}^{*}$ or $Q_{osb3}^{*}$, respectively. The points $(N_{fert}^{osb1}, Q_{osb1}^{*})$ and $(N_{fert}^{osb3}, Q_{osb3}^{*})$ are steady state solutions. $N_{fert}^{osb1}$ and $N_{fert}^{osb3}$ are calculated by rearranging the equation of motion presented in Equation (3) and by substituting $Q_{osb1}^{*} = \bar{Q}_{ur}$ and $Q_{osb3}^{*} = \bar{Q}_{ur}/\nu$ respectively in the rearranged Equation (3). Solving for $N_{fert}^{osb}, j = 1,3$ we get:

$$\begin{equation}
N_{fert}^{osb j} = -\frac{R \cdot Q_R}{(1 - \eta) \cdot 1000} + \frac{[W_p - \alpha \cdot \bar{W}_{ir}]}{\alpha \cdot (1 - \eta) \cdot 1000} \cdot Q_{osb j}, \quad j = 1,3
\end{equation}
$$

D. Mathematical modeling of solving the possible point $(N_{fert}^{osb2}, Q_{osb2})$ of the optimal social benefit, which falls in the continuous and differential open interval of the treatment cost function

The possible point $(N_{fert}^{osb2}, Q_{osb2})$ of the optimal social benefit, which falls in the continuous and differential open interval of the treatment cost function is solved in a dynamic optimization of net present-value social benefit in the framework of a continuous-time optimal control model, and can be stated as:

$$\begin{equation}
\text{MAX} \int_{0}^{\infty} e^{-\tau t} \{P_Y \cdot Y(N_{fert}(t), \bar{W}_{ir}) - TPC(N_{fert}(t), \bar{W}_{ir}) - TTC(Q(t), \bar{Q}_{ur}, \bar{W}_{ur})\} \, dt
\end{equation}
$$

Subject to

$$\dot{Q}(t) = \frac{\alpha \cdot R \cdot Q_R}{W} + \frac{\alpha \cdot \bar{W}_{ir} - W_p}{W} \cdot Q(t) + \frac{\alpha \cdot 1000 \cdot (1 - \eta)}{W} \cdot N_{fert}(t)$$

The current value Hamiltonian for the empirical model is thus given by:

$$H(\cdot) = P_Y \cdot [ - a_0 + a_1 \cdot N_{fert}(t) + a_2 \cdot \bar{W}_{ir} - a_5 \cdot (N_{fert}(t))^{1.5} - a_4 \cdot \bar{W}_{ir}^{1.5} - a_5 \cdot N_{fert}(t) \cdot \bar{W}_{ir}]$$

$$- P_N \cdot N_{fert}(t) - P_W \cdot \bar{W}_{ir} - FC - [(a + b \cdot Q(t)) \cdot (1 - z) \cdot \bar{W}_{ur}]$$

$$+ \lambda(t) \cdot \left\{ \frac{\alpha \cdot R \cdot Q_R}{W} + \frac{\alpha \cdot \bar{W}_{ir} - W_p}{W} \cdot Q(t) + \frac{\alpha \cdot 1000 \cdot (1 - \eta)}{W} \cdot N_{fert}(t) \right\}$$

The three first-order conditions (FOC) necessary for maximum of the current value Hamiltonian are: $\partial H/\partial N_{fert}(t) = 0, \dot{\lambda} = r \cdot \lambda(t) - \partial H/\partial \dot{Q}(t)$ and $\dot{Q}(t) = \partial H/\partial \lambda(t)$. Based on these three conditions, we can characterize a steady state using two isoclines: $N_{fert}(t) = 0$ and $Q(t) = 0$ that are given by:

$$\begin{equation}
N_{fert} \left|_{N_{fert}(t)=0 \atop W(t)=0} \right. = \left[ \frac{P_Y \cdot [a_1 - a_5 \cdot \bar{W}_{ir} - P_N]}{P_Y \cdot 1.5 \cdot a_3} - \alpha \cdot (1 - \eta) \cdot 1000 \cdot b \cdot (1 - z) \cdot \bar{W}_{ur} \right]^2 \cdot \frac{P_Y \cdot 1.5 \cdot a_3 \cdot [r \cdot W + W_p - \alpha \cdot \bar{W}_{ir}]}{P_Y \cdot 1.5 \cdot a_3 \cdot [r \cdot W + W_p - \alpha \cdot \bar{W}_{ir}]} \right)$$

$$\end{equation}$$
and

\[ N_{fert}^{(Q(t) = 0)} = - \frac{R \cdot Q_R}{(1 - \eta) \cdot 1000} + \frac{[\bar{W}_p - \alpha \cdot \bar{W}_{ir}]}{\alpha \cdot (1 - \eta) \cdot 1000} \cdot Q(t) \] (D4)

Equation (D3) satisfies different combinations of \( N_{fert}(t) \) and \( Q(t) \) for \( N_{fert}(t) = \bar{W}(t) = 0 \). But \( N_{fert}(t) \) in Equation (D3) does not depend on \( Q(t) \) (a private case) because the marginal treatment cost function of drinking water is constant for \( Q(t) \) so this variable vanishes in the solution to the second-order condition of the current value Hamiltonian. Equation (D4) is linear with a negative intercept and positive slope and satisfies different combinations of \( N_{fert}(t) \) and \( Q(t) \) for \( \dot{Q}(t) = \bar{W}(t) = 0 \), which converge to a steady state. This equation, developed from the equation of motion presented in Equation (3), could be solved for \( Q(t) \) for any given value of \( N_{fert}(t) \). However, in an economic model, as in this case, \( N_{fert}(t) \) is derived from economic optimization that takes into account the agricultural net private benefit per hectare and the drinking-water treatment cost function. These arguments are optimally considered in solving the two first-order conditions of the current value Hamiltonian (Equation D3). The negative intersect and the positive slope of Equation (D4) cause the two isoclines, Equations (D3) and (D4), to intersect and yield the second possible point of optimal social benefit quantity of nitrogen fertilizer applied and a concentration of nitrogen in groundwater respectively (\( N_{fert}^{obs2} \), \( Q^{obs2} \)), where \( N_{fert}(t) = Q(t) = \bar{W}(t) = 0 \).

\[ N_{fert}^{obs2} = \left[ \frac{P_Y \cdot [a_1 - a_5 \cdot \bar{W}_{ir}] - P_N}{P_Y \cdot 1.5 \cdot a_3} - \frac{\alpha \cdot (1 - \eta) \cdot 1000 \cdot b \cdot (1 - z) \cdot \bar{W}_{ur}}{P_Y \cdot 1.5 \cdot a_3 \cdot [r \cdot W + W_p - \alpha \cdot \bar{W}_{ir}]} \right]^2 \] (D5)

and

\[ Q^{obs2} = \frac{\alpha \cdot R \cdot Q_R}{W_p - \alpha \cdot \bar{W}_{ir}} + \left( \frac{\alpha \cdot (1 - \eta) \cdot 1000}{W_p - \alpha \cdot \bar{W}_{ir}} \right) \cdot \left[ \frac{P_Y \cdot [a_1 - a_5 \cdot \bar{W}_{ir}] - P_N}{P_Y \cdot 1.5 \cdot a_3} - \frac{\alpha \cdot (1 - \eta) \cdot 1000 \cdot b \cdot (1 - z) \cdot \bar{W}_{ur}}{P_Y \cdot 1.5 \cdot a_3 \cdot [r \cdot W + W_p - \alpha \cdot \bar{W}_{ir}]} \right]^2 \] (D6)

E. The mathematical condition for maximizing the welfare change

The mathematical condition for maximizing the welfare change is based on the social planner optimal condition in steady state. This condition is presented in Equation (E1) – developed by rearranging Equation (D3) in Section D above – for the continuous and differential open interval of the treatment cost function:

\[ \frac{P_Y \cdot (a_1 - 1.5 \cdot a_3 \cdot (N_{fert}(t))^{0.5} - a_5 \cdot \bar{W}_{ir})}{VMP} = \frac{P_N}{MPC} + \frac{\alpha \cdot (1 - \eta) \cdot 1000 \cdot b \cdot (1 - z) \cdot \bar{W}_{ur}}{r \cdot W(t) - \alpha \cdot \bar{W}_{ir} + W_p} \] (E1)
Welfare change is maximized in steady state where the value of the marginal product (VMP) – the term on the left-hand side of Equation (E1) – is equal to the marginal social cost (MSC) – the term on right-hand side of (E1). This cost includes marginal private cost (MPC) – the first term on the right-hand side – and the marginal treatment cost (MTC) – the second term on the right-hand side. The social planner optimal condition is illustrated also graphically in Figure 1.